Calibration of Dynamic Models under Uncertainty

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Outline

- Overview of research objectives
- Relevant prior research
- Brief introduction of calibration metrics
- Introduce two example problems for vibration and impact dynamics model calibration
  - Example 1: Plate vibration of an orthogrid
  - Example 2: Impact dynamics of Helicopter
- Typical results: Did we get what we were looking for?
- Summary

Background Definitions

Validation- the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model

Verification- the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution of the model

Calibration- Process of adjusting numerical or physical modeling parameters in the computational model for the purpose of improving agreement with experimental data

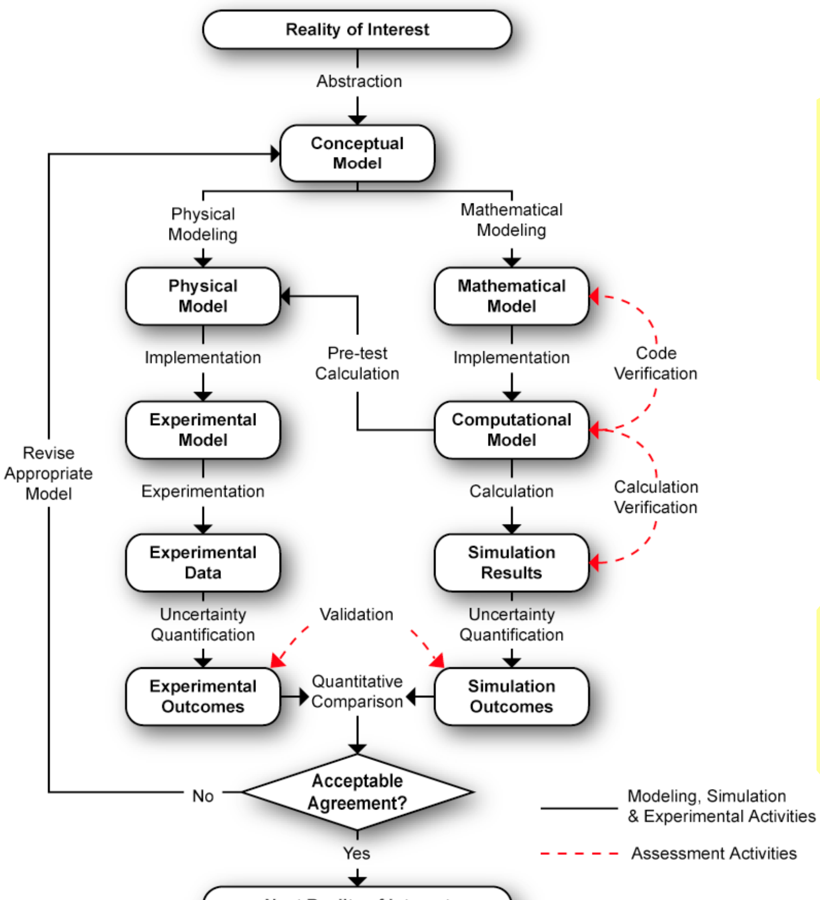
Overview of Research Objectives/Approach

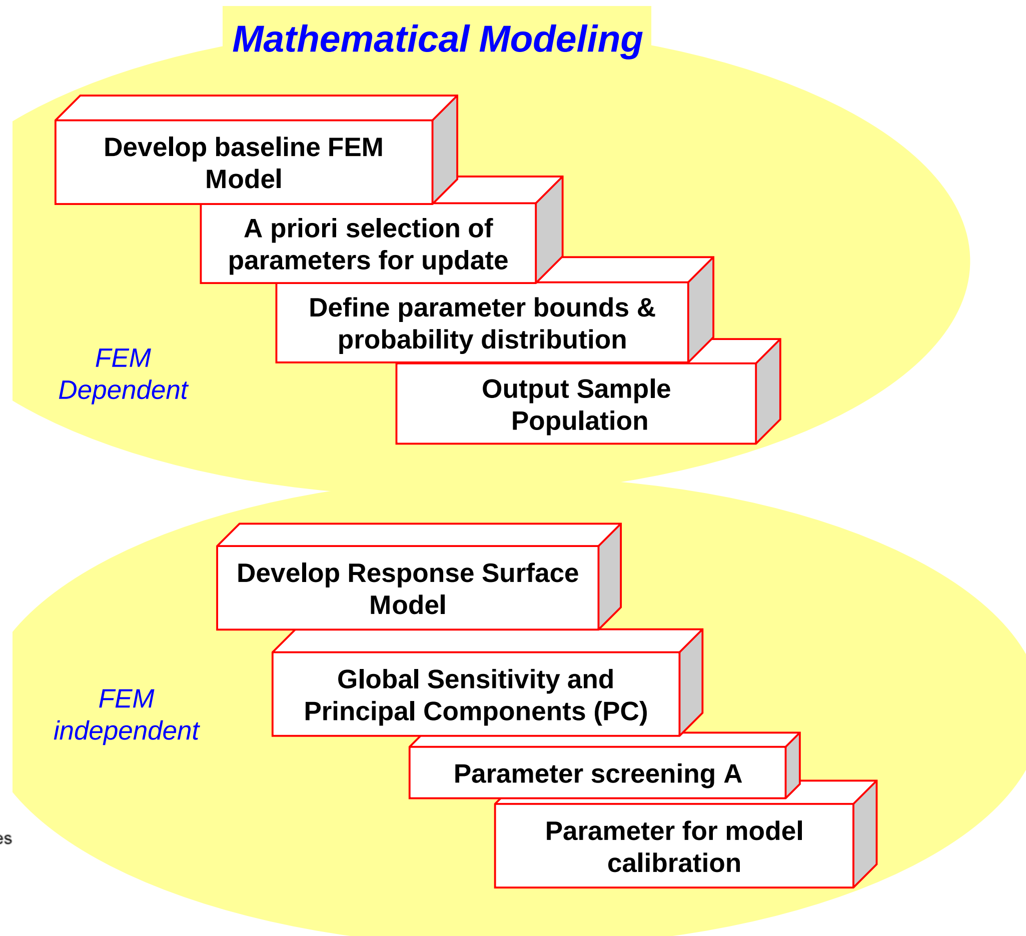
- Technology Driver: NASA's programs are slowly moving towards having probabilistic requirements levied on all hardware producers (e.g. Constellation, AERO)
- Objective: To identify underlying issues related to verifying commonly used metrics that account for uncertainty
  - Understand analysis and test methods
  - Provide guidance on uncertainty quantification assessments (test and analysis)
  - Develop methodology to address requirements as defined by program
- Approach: To develop methods that take advantage of models developed using deterministic approaches to assess uncertainty in the results for linear and non-linear problems
  - Apply new deterministic sampling techniques to develop statistical databases for critical response quantities (e.g., acceleration, stresses, loads, …)
  - Consolidate knowledge gained from limited time-consuming solutions into response surface models for analysis of variance, designs of experiments, and model update
- Product: Verification and validation of models under uncertainty

Relevant Prior Research

- Verification and Validation of models has fostered thousands of papers in the US and abroad
- In September 2006 The Nuclear Regulatory Commission issued for review NUREG-0800, Chapter 19.1 on “Determining the Technical Adequacy of Probabilistic Risk Assessment Results for Risk-Informed Activities” www.nrc.gov
- Today, the Nuclear Regulatory Commission is perhaps the only government agency with established procedures for risk assessment of nuclear reactors using probabilistic requirements
- Technical societies like AIAA, ASME, ANS, SAE and SEM are actively pursuing development of standards for use by practitioners, for example, AIAA Guide for the Verification and Validation of Computational Fluid Dynamics Simulations (G-077-1998)
- In 2006, NASA sponsored the development “Standard for Models and Simulations” NASA-STD-(I)-7009, to address methods of presenting results from simulations to program managers for decision making
- Early in 2007, NASA’s Chief Engineer office appointed a group to provide recommendations on how to write probabilistic requirements for the Constellation Program
- Recent SDB work in the uncertainty quantification area
  - Horta, L.G., Reaves, M.C., Jackson, K.E., Annett, M.S., Littell, J.D.: “Fuselage Impact Testing and Simulation: A Model Calibration Exercise,” Proceedings of the IMAC-XXII A Conference and Exposition on Structural Dynamics, Orlando, FL, February, 2014.
  - Horta, L.G., and Reaves, M.C.: “An Independent Assessment of the SLS-Core State Modal Test,” NASA/TM-2015-0218766, Distribution limited to NASA Personnel and NASA Contractors Only
  - Horta, L.G., and Reaves, M.C.: “A Modal Calibration Challenge Problem Using Simulated SLS Core-Stage Modal Test Data.” NASA/TM-2017-219586, Distribution limited to NASA Personnel and NASA Contractors Only
- State of Practice for uncertainty quantification in the structural dynamics community is not in general well accepted because old habits are hard to break

Research: Computational Approach Towards Model Validation & Verification

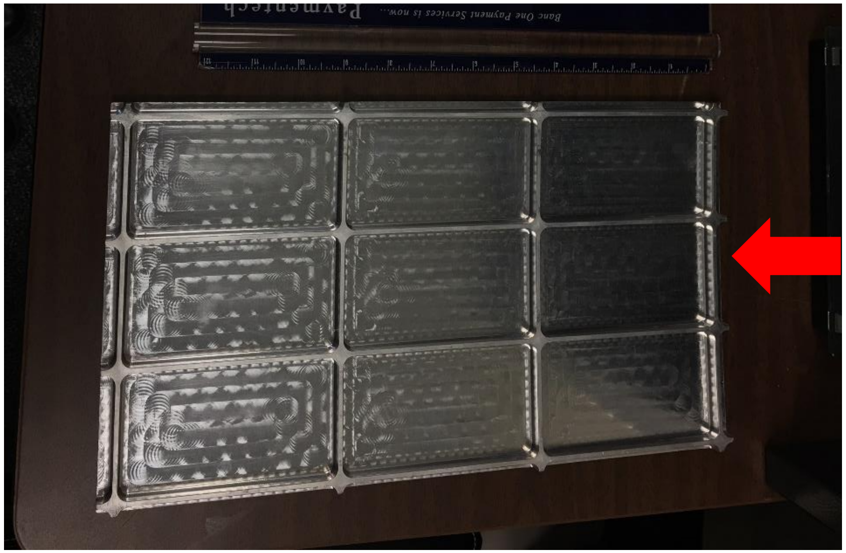




Ref. Rutherford, AIAA 2005-1906

Calibration of vibration models: Orthogrid panel example

Orthogrid Panel Configuration



Example -1: Rectangular orthogrid configuration (8 x 14 1/8 in) Panel thickness 3/32 in (thickness ranged 0.09375-0.098 in), panel depth 15/32 in. Weight 1.35 lbf (612.2 grams)

Triangular orthogrid design frequently used in large panel construction

Error metrics to assess model calibration

- Let $H(f,p)$ be the analysis frequency response function (FRF) for parameter p and frequency f. Also let $\underline{\sigma}(f) = min_{\forall p} H(f,p)$, $\overline{\sigma}(f) = max_{\forall p} H(f,p)$, and $H_t(f)$ be the measured FRF. The probability that the existing model can encompass the test data, given N model realizations, is bounded by

$$PV(f) = Prob\left(\underline{\sigma}(f) \leq H_t(f) \leq \bar{\sigma}(f)\right) \cong (N-1)/N$$

- FRF error metric g_0 compares test to analysis over a range of frequencies $f_i \in \{f_1, f_n\}$ weighted with matrix W

$$g_0 = \sum_{i=1}^{n} tr\left\{(H(f_i,p) - H_t(f_i))^T W_i (H(f_i,p) - H_t(f_i))\right\}$$

- METRIC 1: Modal assurance criterion metric g_1 compares test mode shapes to analysis mode shapes

$$g_1 = \|I - MAC\|_{frob}, \quad MAC \triangleq \left(\frac{\Psi^T \Phi}{\|\Psi\|\|\Phi\|}\right)^2$$

- METRIC 2: Similarly, the orthogonality error metric g_2 compares test modes Ψ to analysis mode shapes Φ using the reduced mass matrix M

$$g_2 = \left\|(I - \Psi^T M \Phi) W\right\|_{frob}$$

- Two variations of these metrics also used are: 1) $J_1 = \log(g_0 g_1)$ and 2) $J_2 = \log(g_0 g_2)$

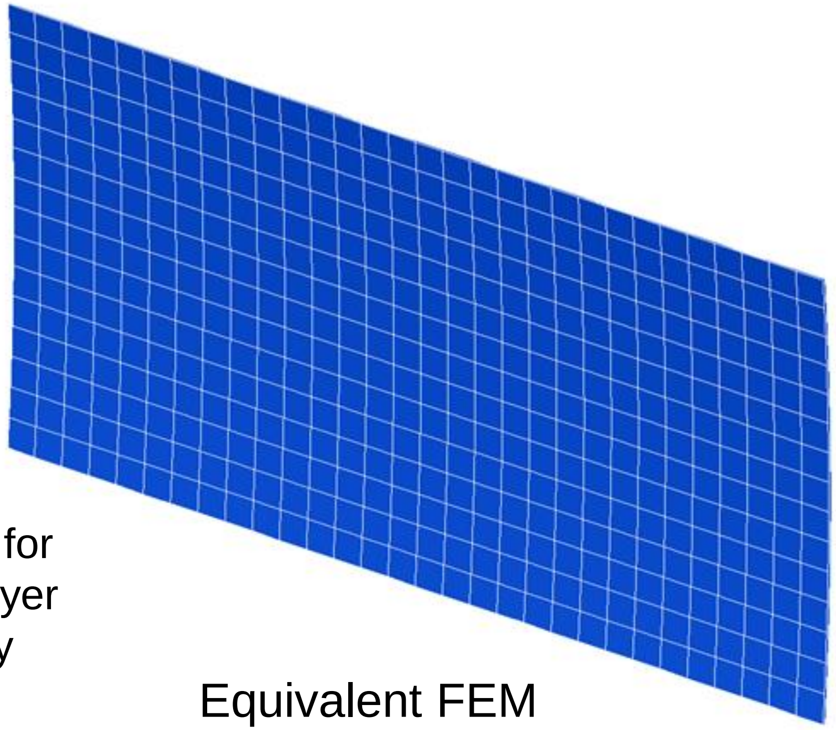
Equivalent-FEM* Model for rectangular panel

Model Equation

$$M\ddot{x} + Kx + c\dot{x} = Bf$$

Model created with MSC NASTRAN

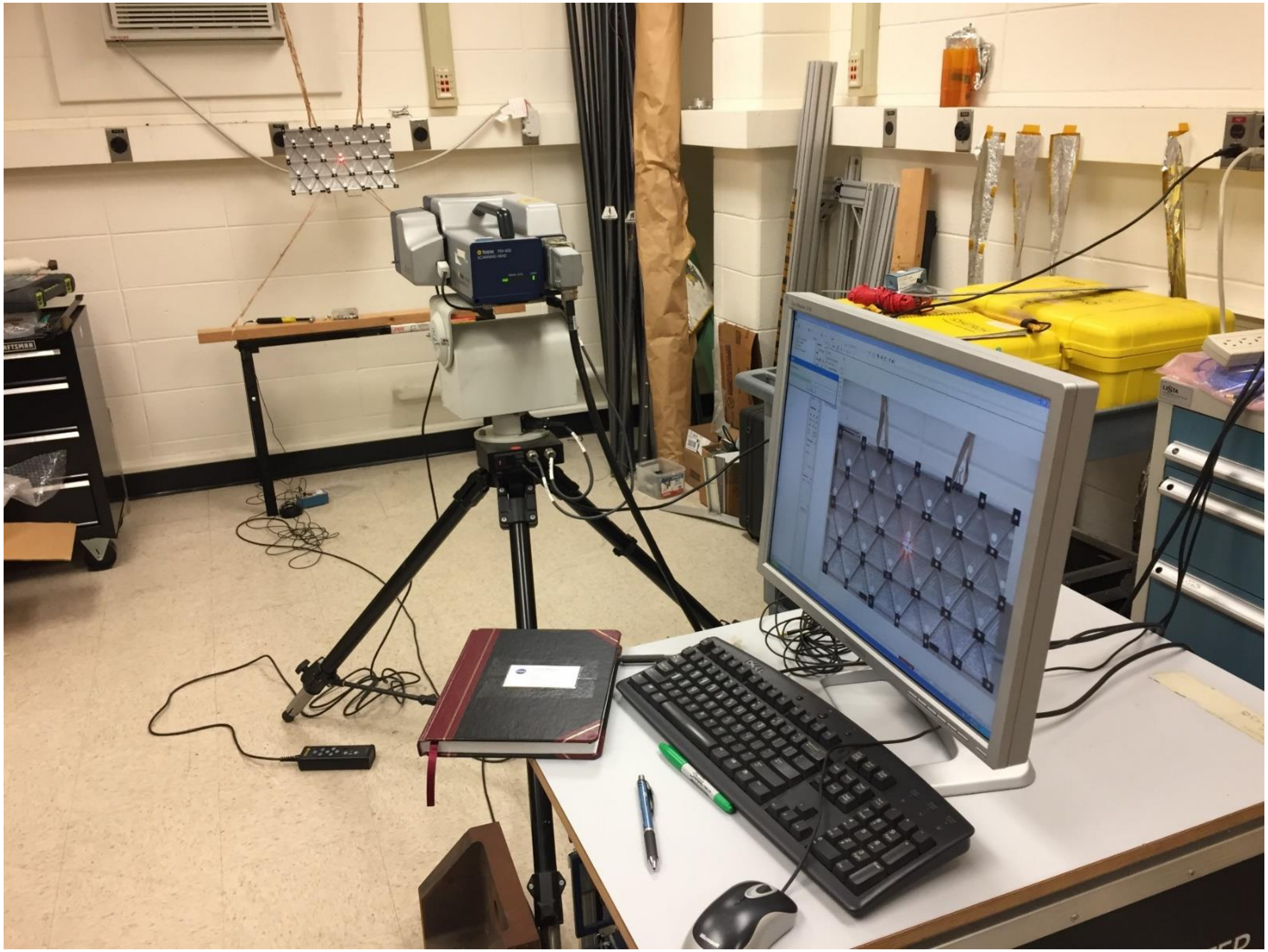
- 450 CQUAD4 elements
- 496 nodes (2976 DOF)
- Skin Element size ~ 0.5”X0.5”
- Composite laminate with isotropic properties for skin layer and orthotropic properties for rib layer
- Mass is distributed between the two layers by adjusting the density of each layer



Equivalent FEM

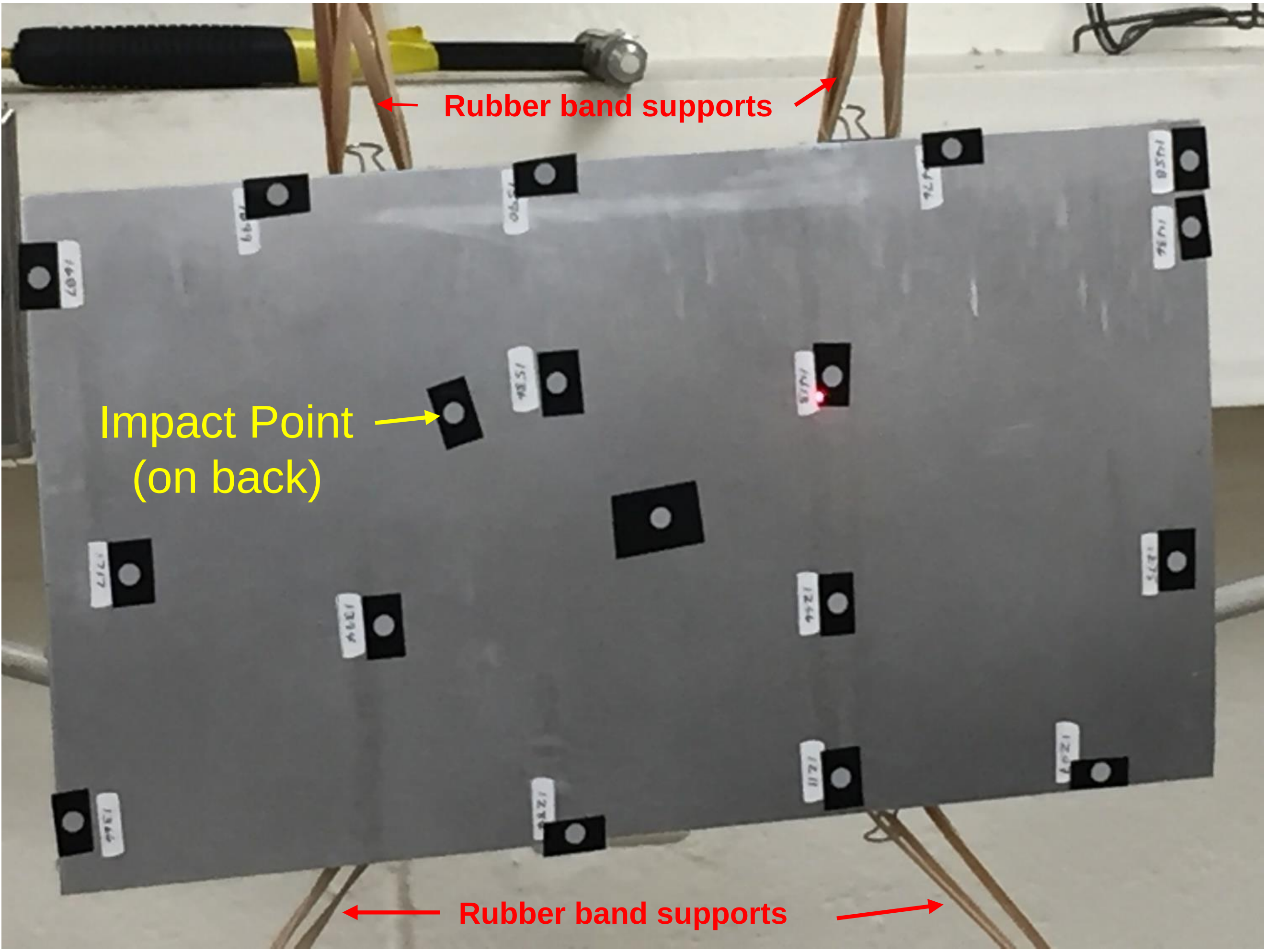
*FEM- Finite Element Model

Vibration Test using Laser Vibrometer



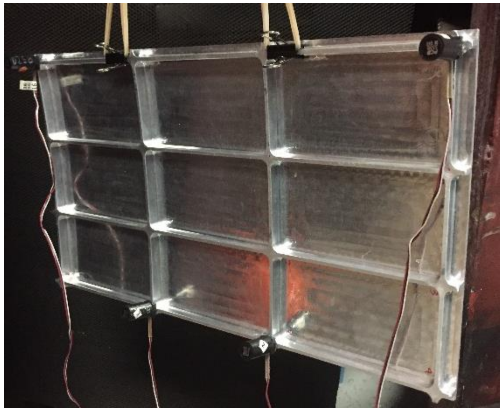
Rectangular orthogrid panel test setup

Front View



- Panel supported at the top and bottom using rubber bands
- Hammer impact on the back
- Target locations selected using IRKE
- 23 locations measured

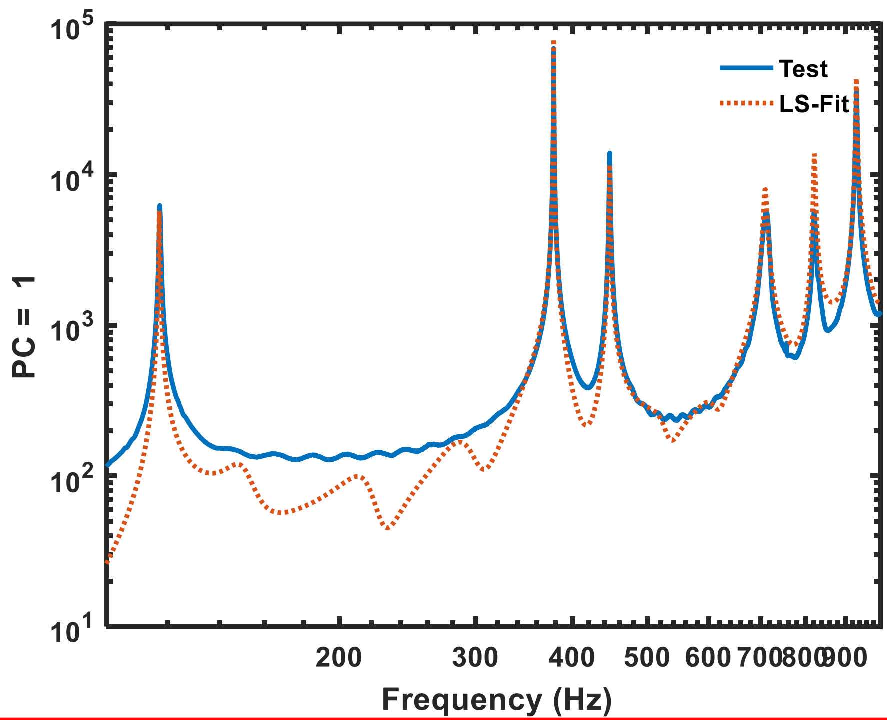
Back View



Modal test results for rectangular panel

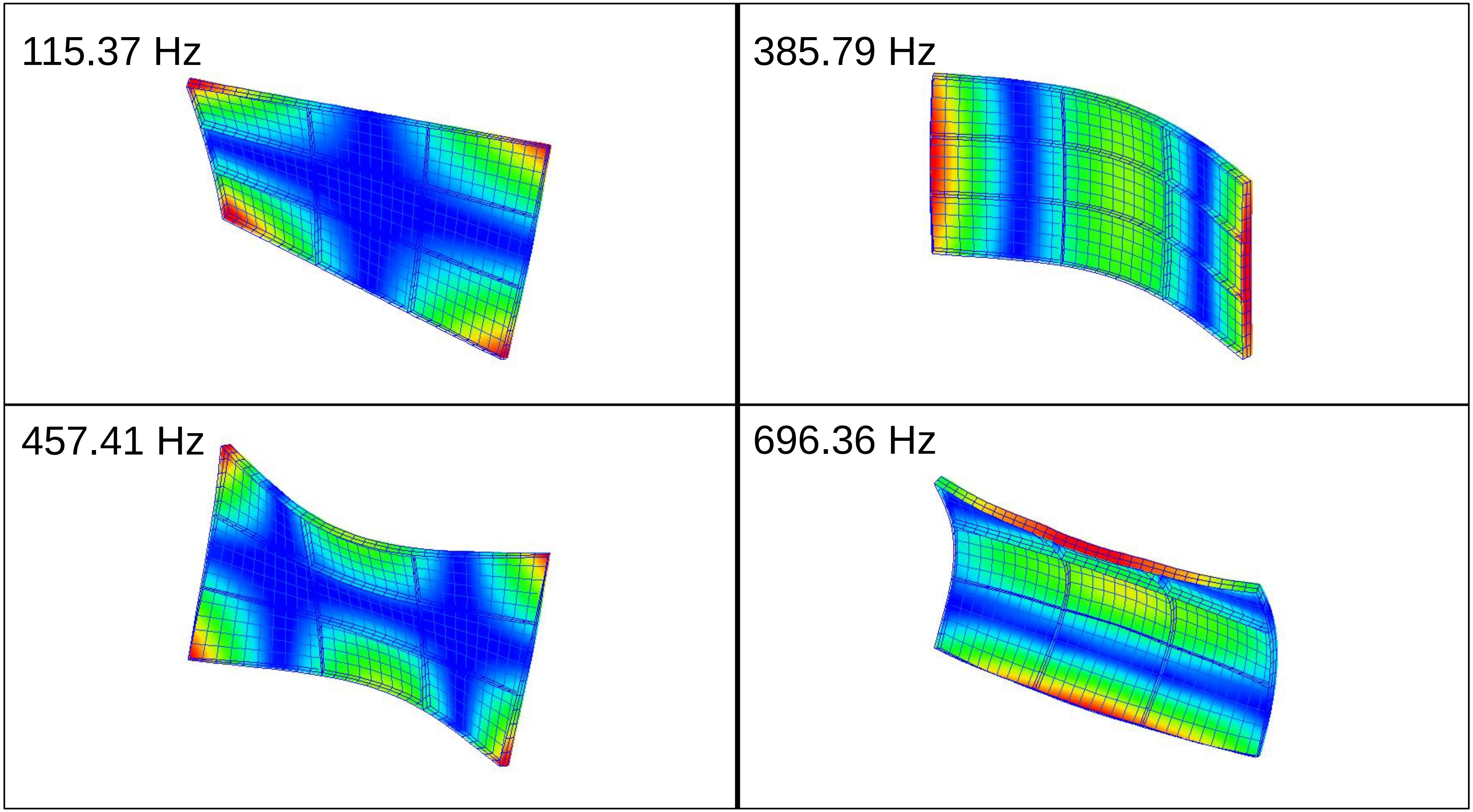
Detail-FEM Freq. (Hz)	Equiv-FEM Freq. (Hz)	ERA-ID Freq. (Hz)	Detail-FEM Error (%)	Equiv-FEM Error (%)
115.4	107.7	117.0	-1.4	-7.9
385.8	359.2	378.3	1.9	-5.0
457.4	392.5	446.8	2.3	-12.2
696.4	767.8	709.6	-1.9	8.2
813.3	795.7	821.5	-1.0	-3.1
918.0	948.5	930.8	-1.4	1.9

*Eigensystem Realization Algorithm (ERA) - is used to recover mode shapes, frequencies, and damping values

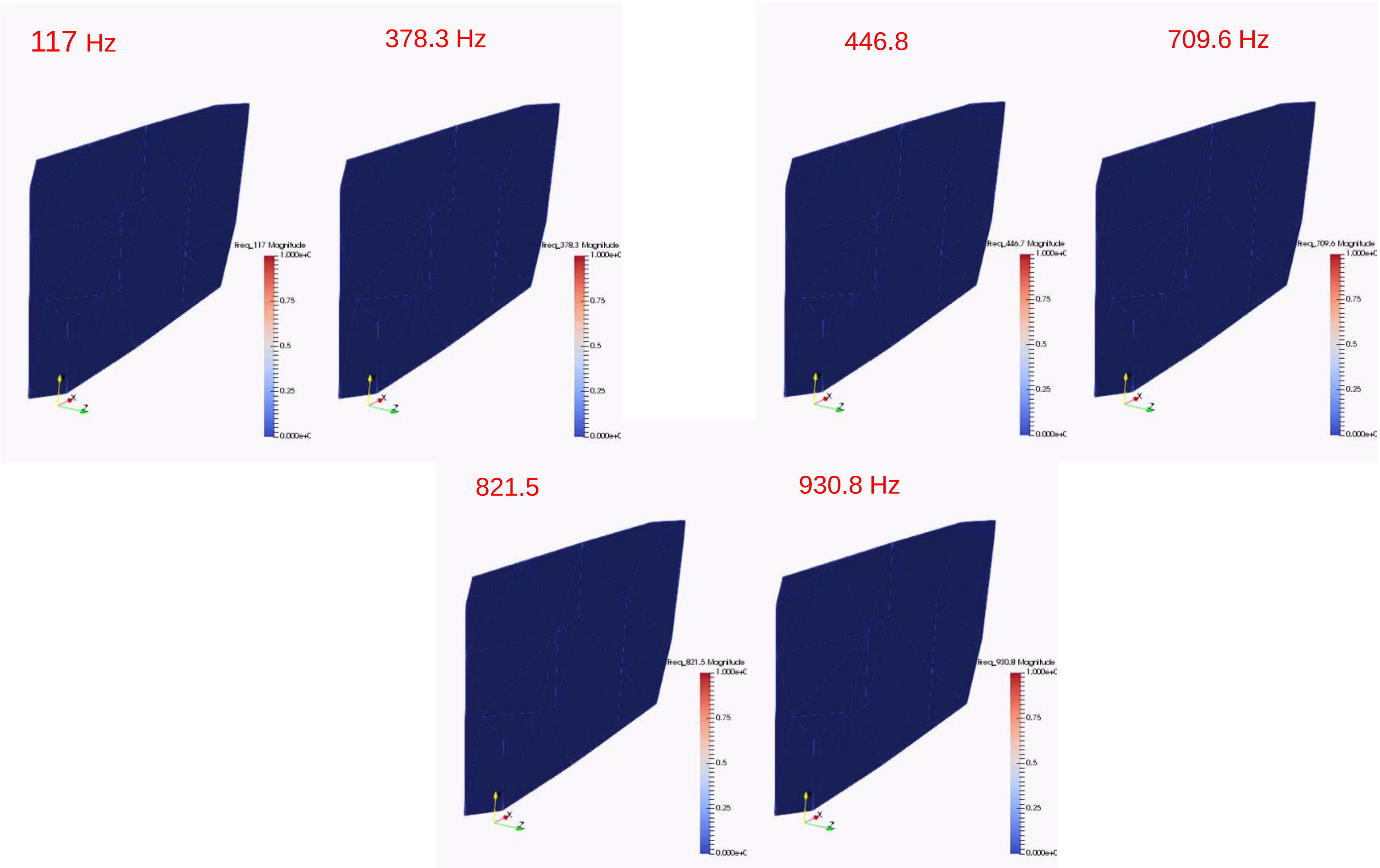


Principal Value curve fit ERA fit Vs FRF Data

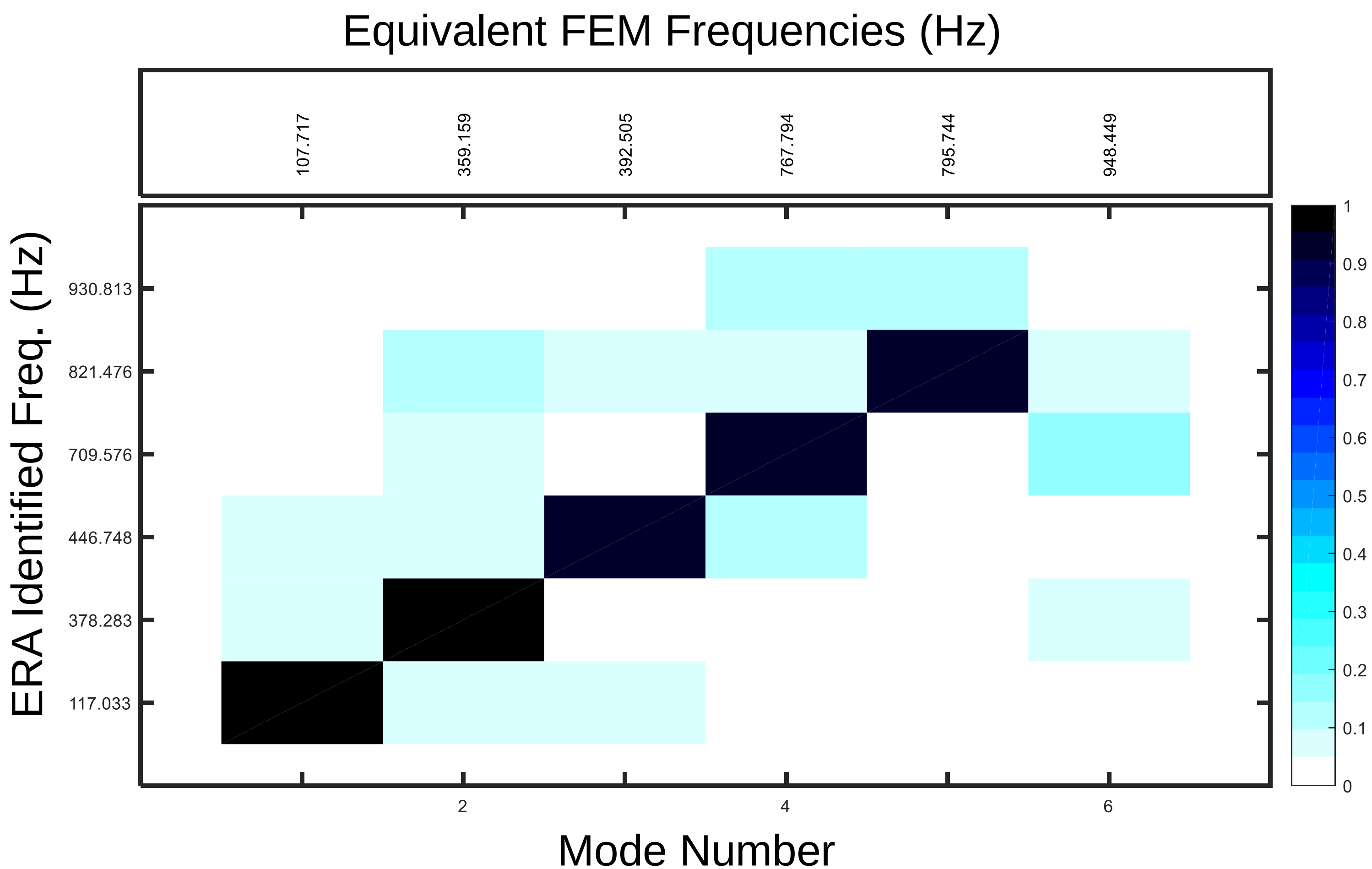
Analytically predicted panel modes



Measured rectangular orthogrid panel modes



METRIC 2: Orthogonality of ERA vs Equivalent-FEM (23 sensors)



What was learned from this test?

- Analytically predicted frequencies were up to 12.2% lower that the measured values
- Mode shape orthogonality error ~ 20%
- QUESTION: How do we calibrate the model based on what we learned from test?

ANSWER: There are many approaches but not one that is universally accepted

Calibration of Impact Dynamic Models
Example 2: Helicopter model

Multi-dimensional calibration of Impact Models

Objectives

- To define a process to conduct systematic model calibration of impact dynamic models to improve the multi-dimensional predictive capability of models
- To develop pre-test analysis guidance to maximize impact of test data for model calibration

Calibration Metrics

- Metric 1: Let $Q(t,p)=\|v\|_2$ be the 2-norm of a response vector v and let $\bar{\sigma}=\max_{\forall p} Q(t,p)$ and $\underline{\sigma}=\min_{\forall p} Q(t,p)$. The probability to be able to reconcile test with analysis given N model realizations is bounded by

$$M_1 = Prob(\underline{\sigma} \le Q_e^*(t) \vee Q_e(t) \ge \bar{\sigma}) << 1/N$$

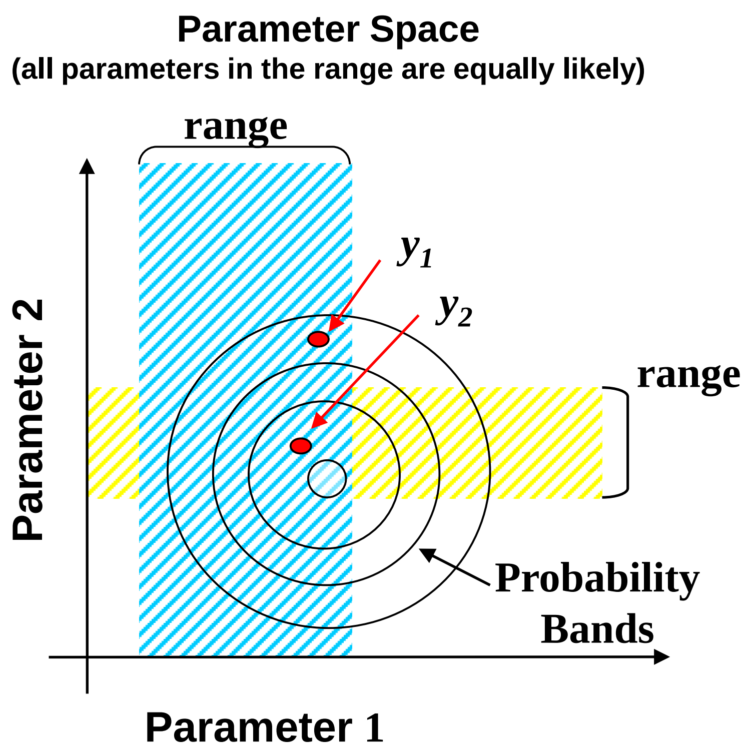
- Metric 2: Given a set of measured impact shapes as $\breve{\Gamma}$ and predicted impact shapes $\breve{\Sigma}$ the orthogonality metric compares multi-dimensional closeness of test and analysis

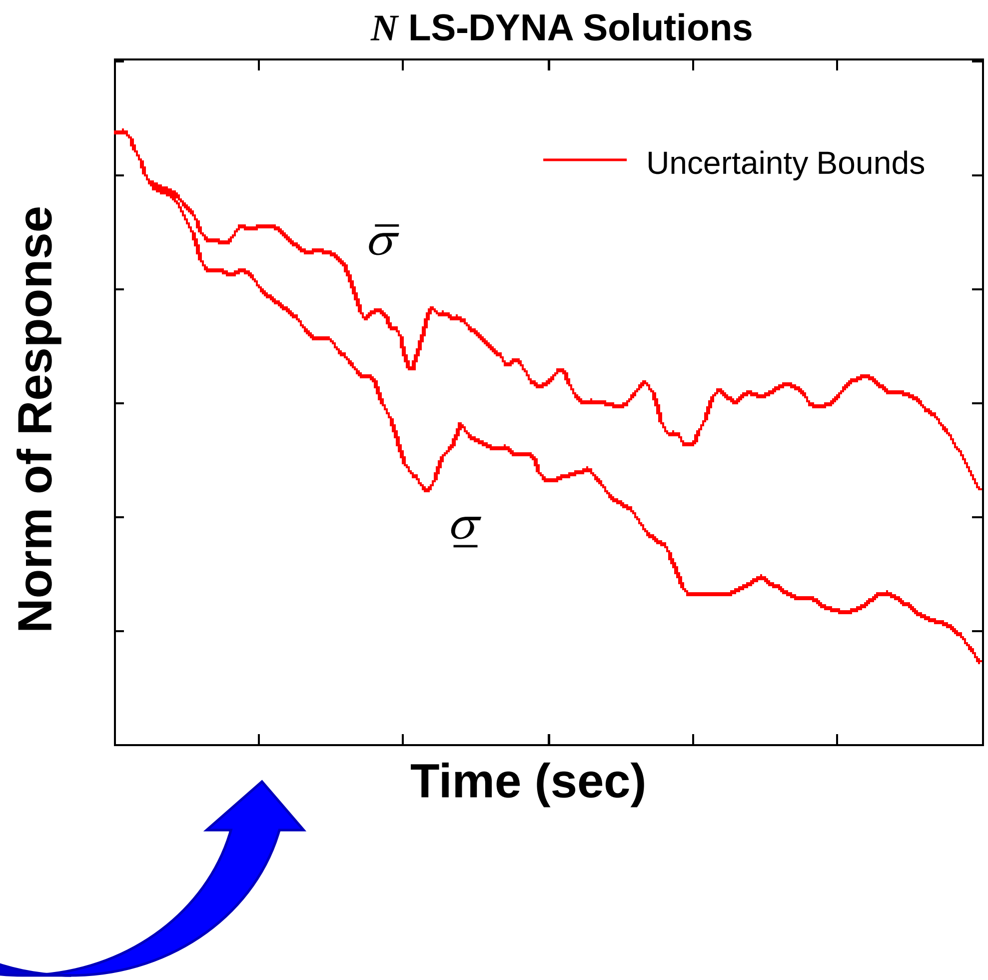
$$M_2 = \breve{\Gamma}^T \breve{\Sigma}$$

"Good" metric values indicated by M_2~I

* $Q_e(t)$ is the vector 2-norm with experimental data

How is Parameter Uncertainty Accounted for?

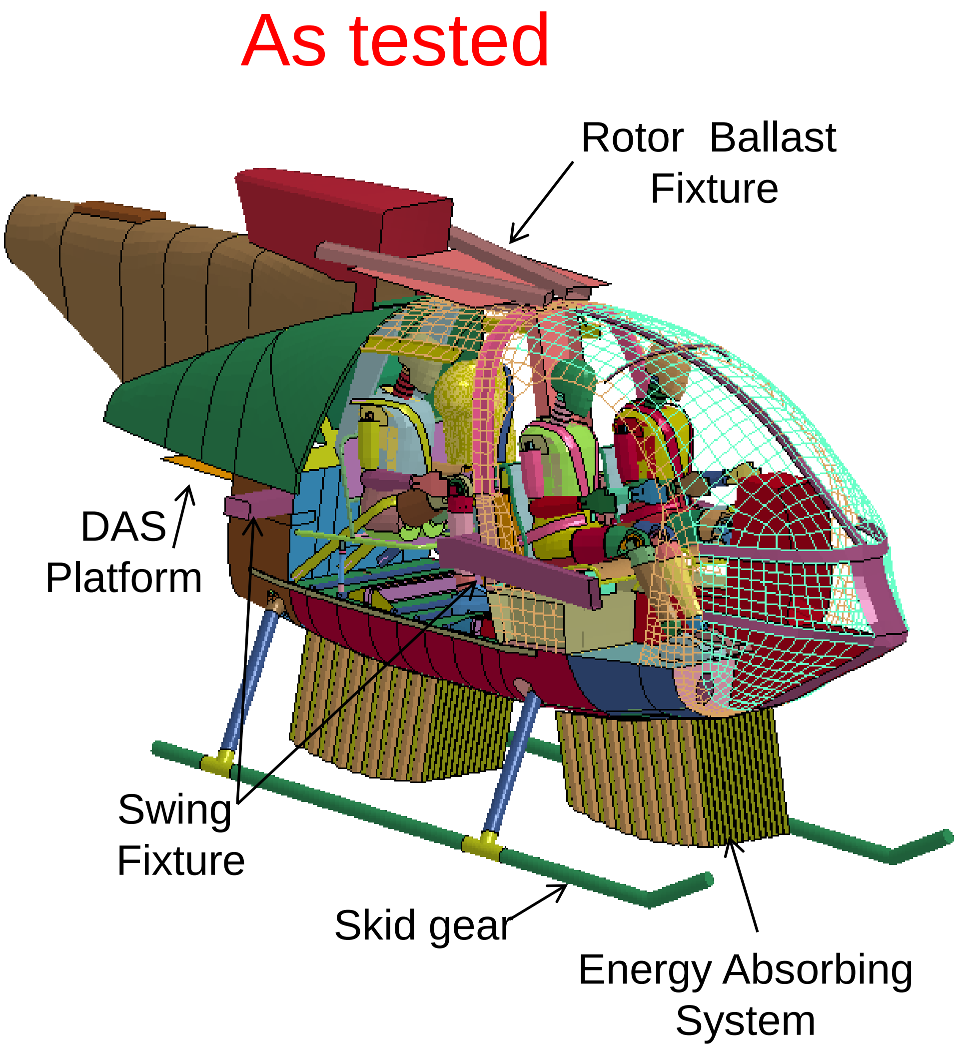


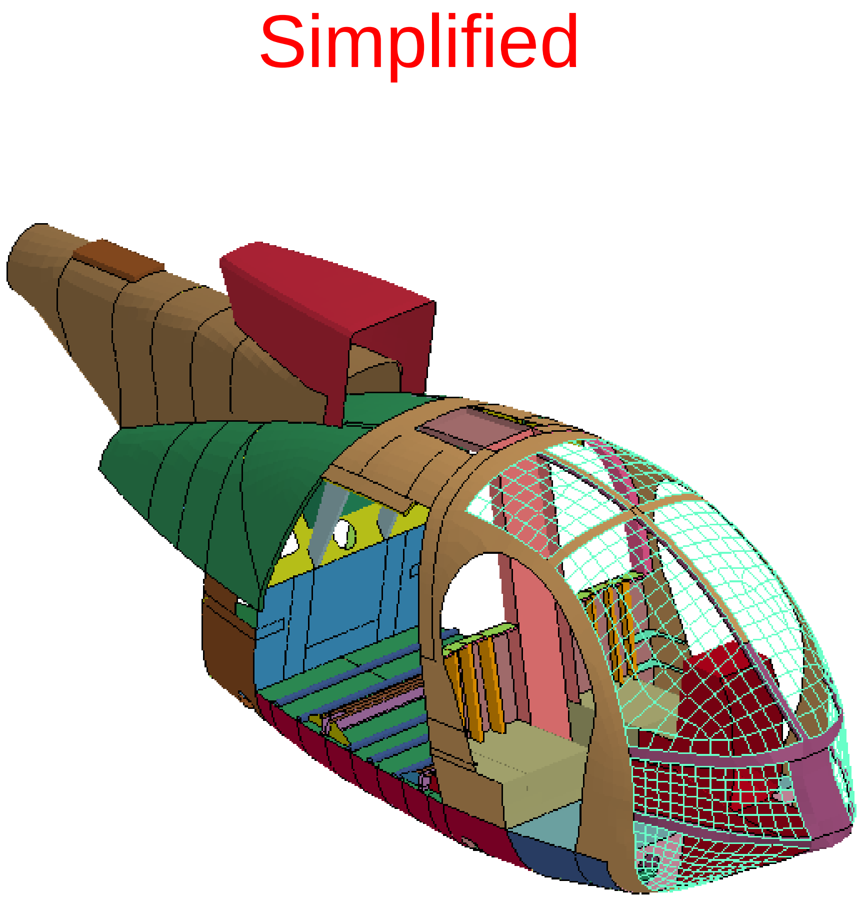


Calibration Demonstration Example

- To demonstrate the process the MD-500 helicopter model parameters were altered and alterations were concealed
- Simulated data using the altered model was used in lieu of experimental data
- Calibration process was applied to reconcile model with test

Simplified Helicopter FEM for Calibration Demonstration





Impact Shape Formulation

- General system response time history decomposition:

measured

unmeasured

$$\begin{pmatrix} y(t) \\ y_e(t) \end{pmatrix} = \sum_{i=1}^{n} \begin{bmatrix} \phi_i \\ \psi_i \end{bmatrix} \sigma_i g_i(t) = \begin{bmatrix} \Phi \\ \Psi \end{bmatrix} \Sigma G(t)$$

- Expansion relationship:

$$y_e(t) = \Psi \left(\Phi^T \Phi \right)^{-1} \Phi^T y(t)$$

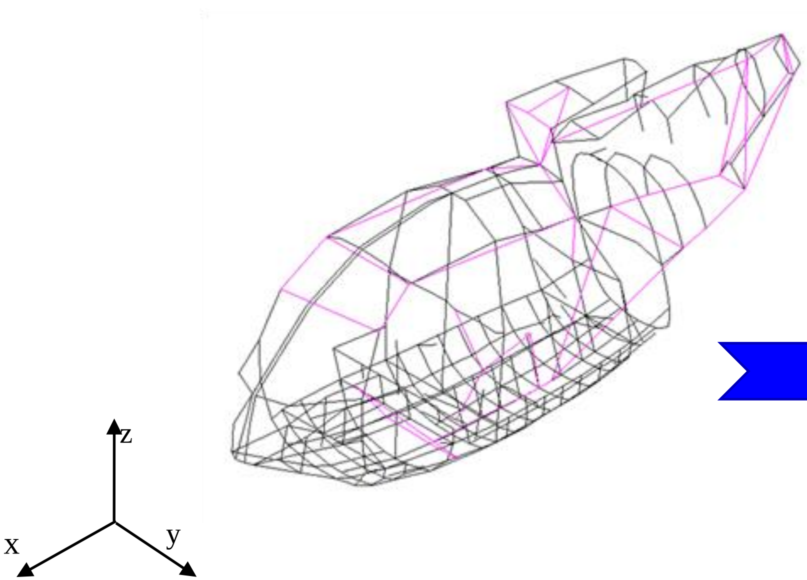
LS-DYNA Impact Shapes

Measured Impact Shapes

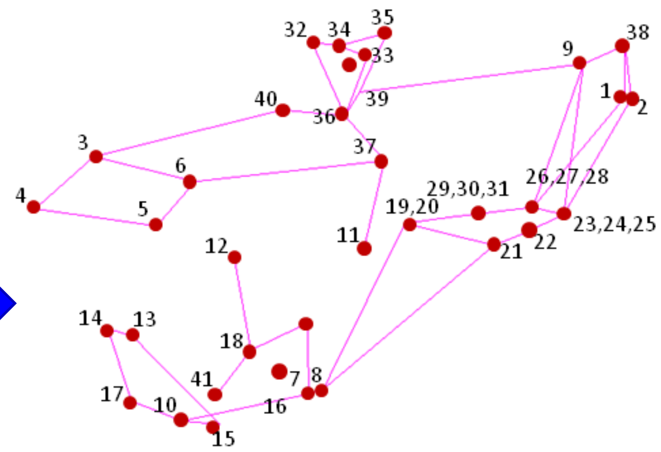
- Impact shape contribution to response

$$\delta_i = \sigma_i / \sum_{l=1}^{n} \sigma_l$$

Optimal Sensor Placement for Simplified Model

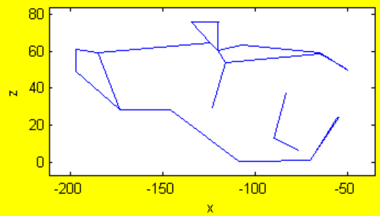


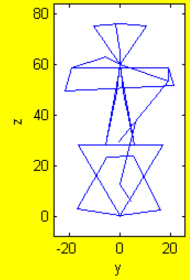
Expanded geometry for Simplified Model

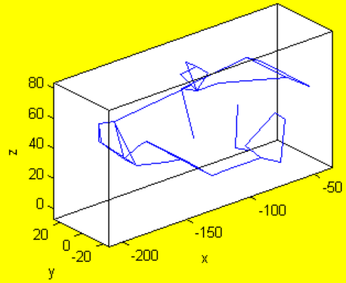


41 sensors used for Simulated Experiment

Comparison of Impact Shapes in Test and Expanded Coordinates



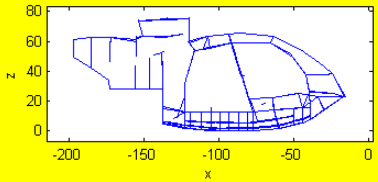


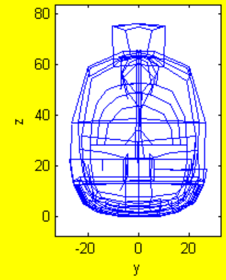


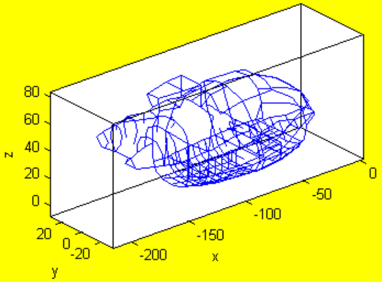
Impact Shape No. 1
Contribution 37 %

Reduced

Expanded

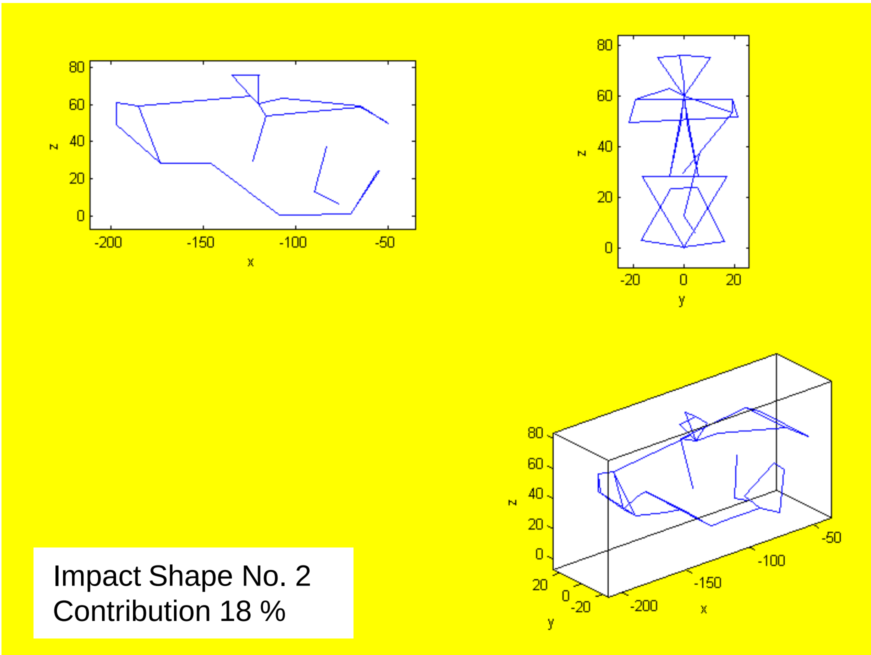






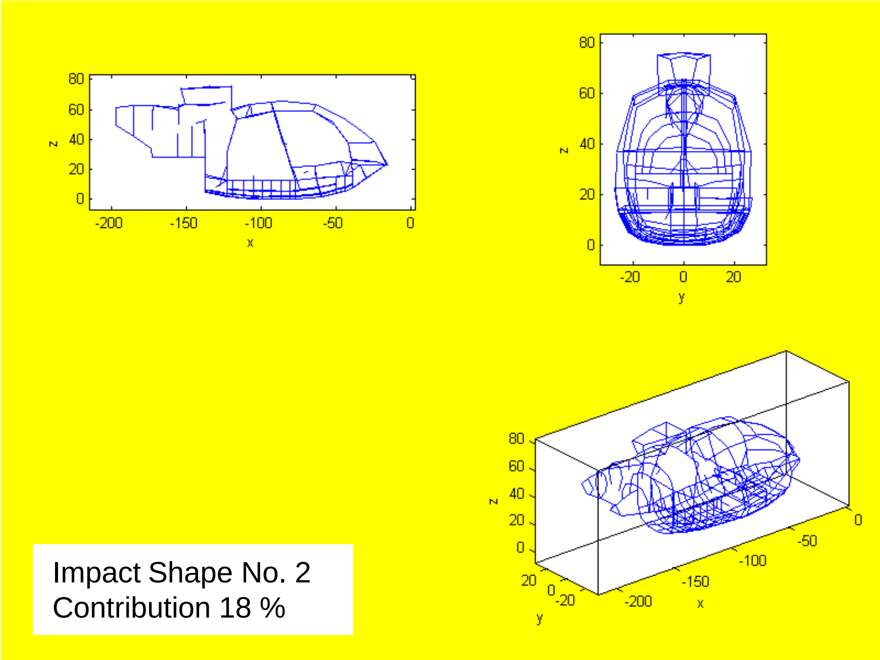
Impact Shape No. 1
Contribution 37 %

Comparison of Impact Shapes in Test and Expanded Coordinates



Reduced

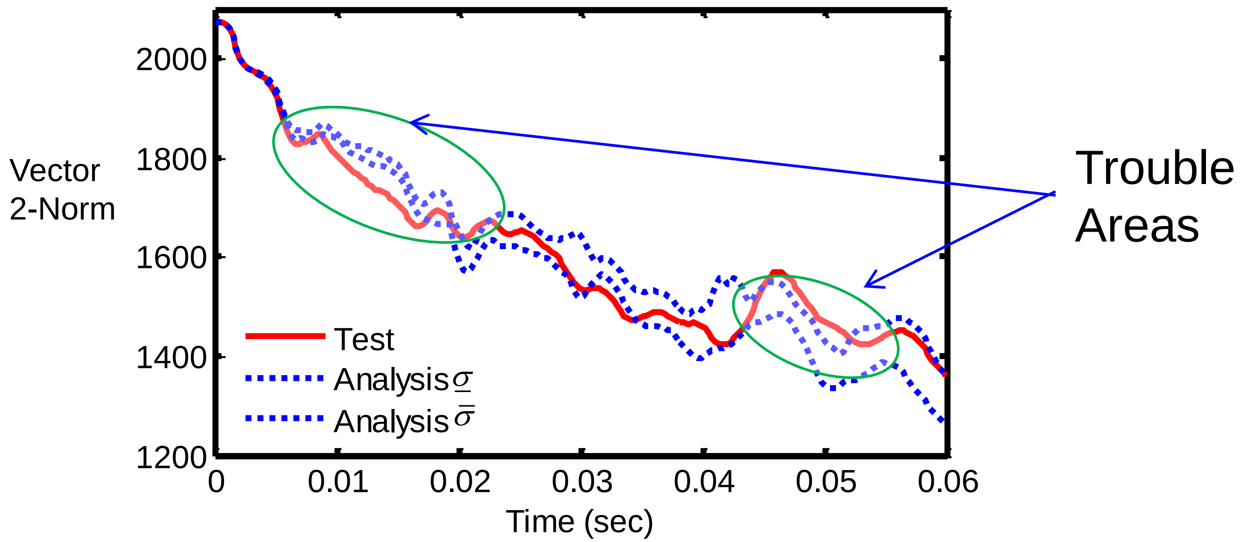
Expanded



Results for Helicopter Calibration Example 1st Attempt

Initial Parameter Selection for Simplified Helicopter Model

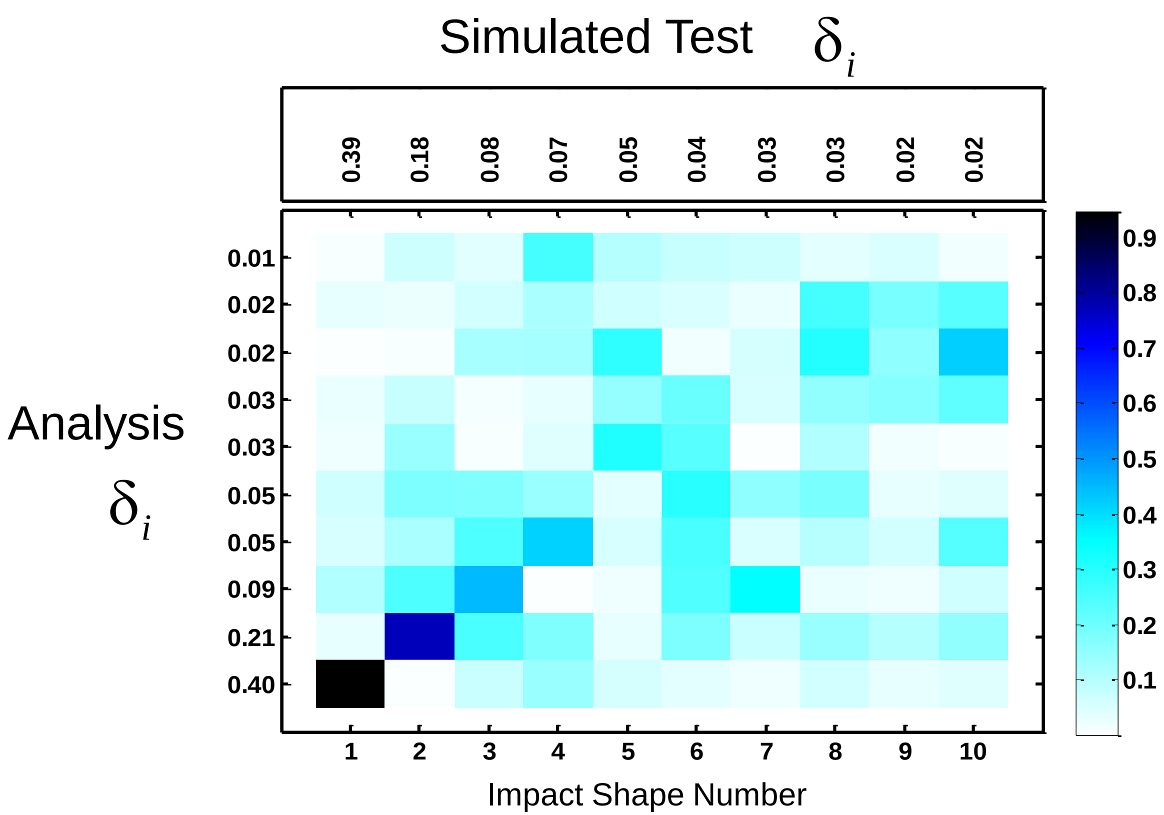
Metric 1



No.	Parameter Description	Nominal	Lower Bound	Upper Bound
1	E back panel (lbs/in^2)	10,000,000	8,000,000	12,000,000
2	E subfloor ribs (lbs/in^2)	10,000,000	8,000,000	12,000,000
3	E keel beam web (lbs/in^2)	9,880,000	7,904,000	11,856,000
4	E stinger upper tail (lbs/in^2)	10,000,000	8,000,000	12,000,000
5	E stinger lower tail (lbs/in^2)	10,000,000	8,000,000	12,000,000
6	Et subfloor ribs (lbs/in^2)	134,200	107,360	161,040
7	Et keel beam web (lbs/in^2)	134,200	107,360	161,040
8	Et lower tail stinger (lbs/in^2)	134,200	107,360	161,040

Orthogonality results with Nominal Parameter

Metric 2

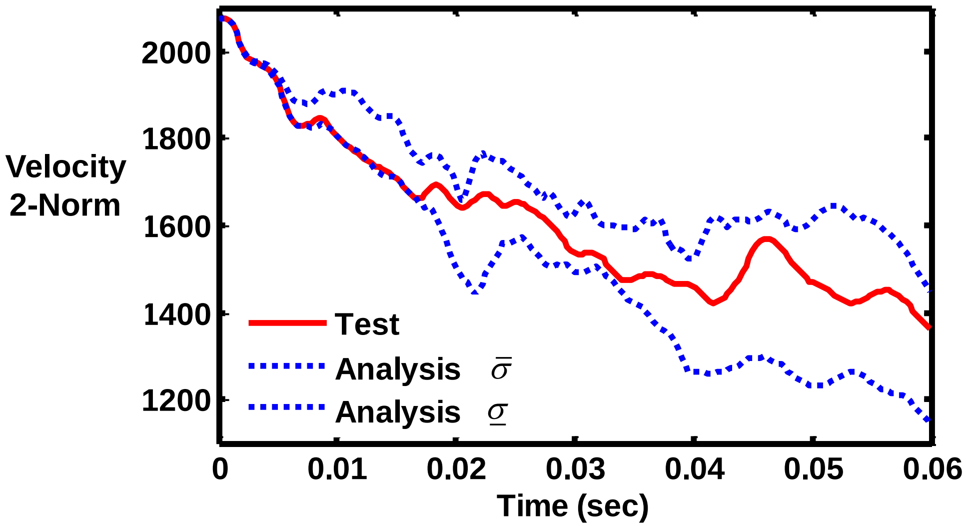


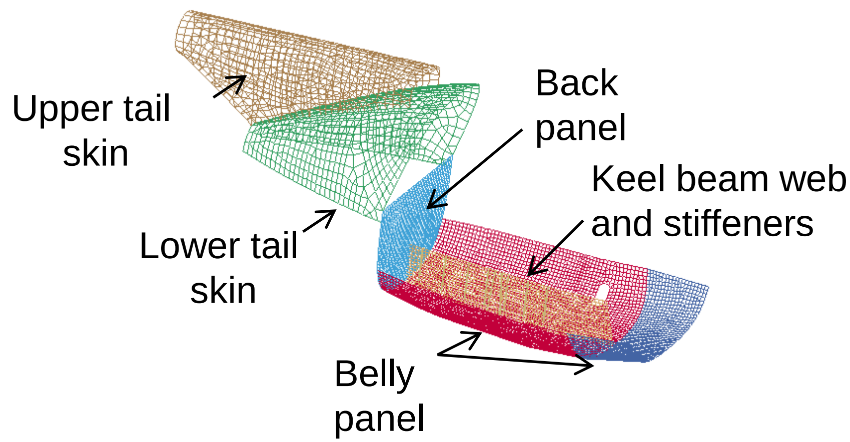
δ_i Impact shape contribution to total response

Results for Helicopter Calibration 2nd Attempt

Revised Parameter Selection for Simplified Helicopter Model

Metric 1



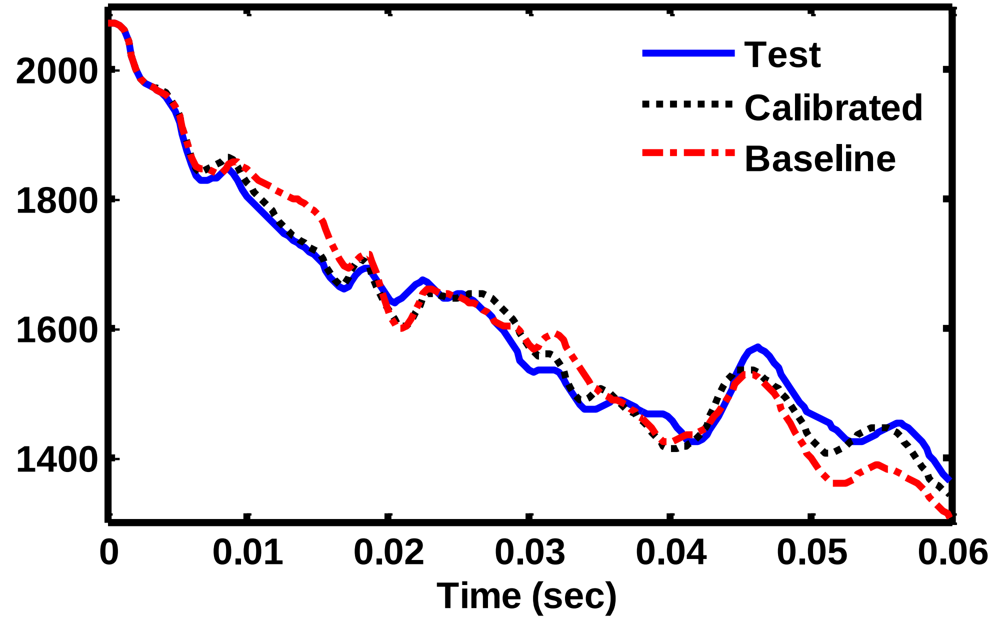


No.	Parameter Description	Nominal	Lower Bound	Upper Bound
1	Keel beam stiffener thickness (in)	0.020	0.015	0.025
2	Belly panel thickness (in)	0.090	0.08	0.135
3	Keel beam thickness (in)	0.040	0.035	0.045
4	Lower tail thickness (in)	0.040	0.035	0.045
5	Back panel thickness (in)	0.020	0.015	0.025
6	Upper tail thickness (in)	0.020	0.015	0.025

Comparison of Metric 1 Results for Baseline, Calibrated and Test

Velocity

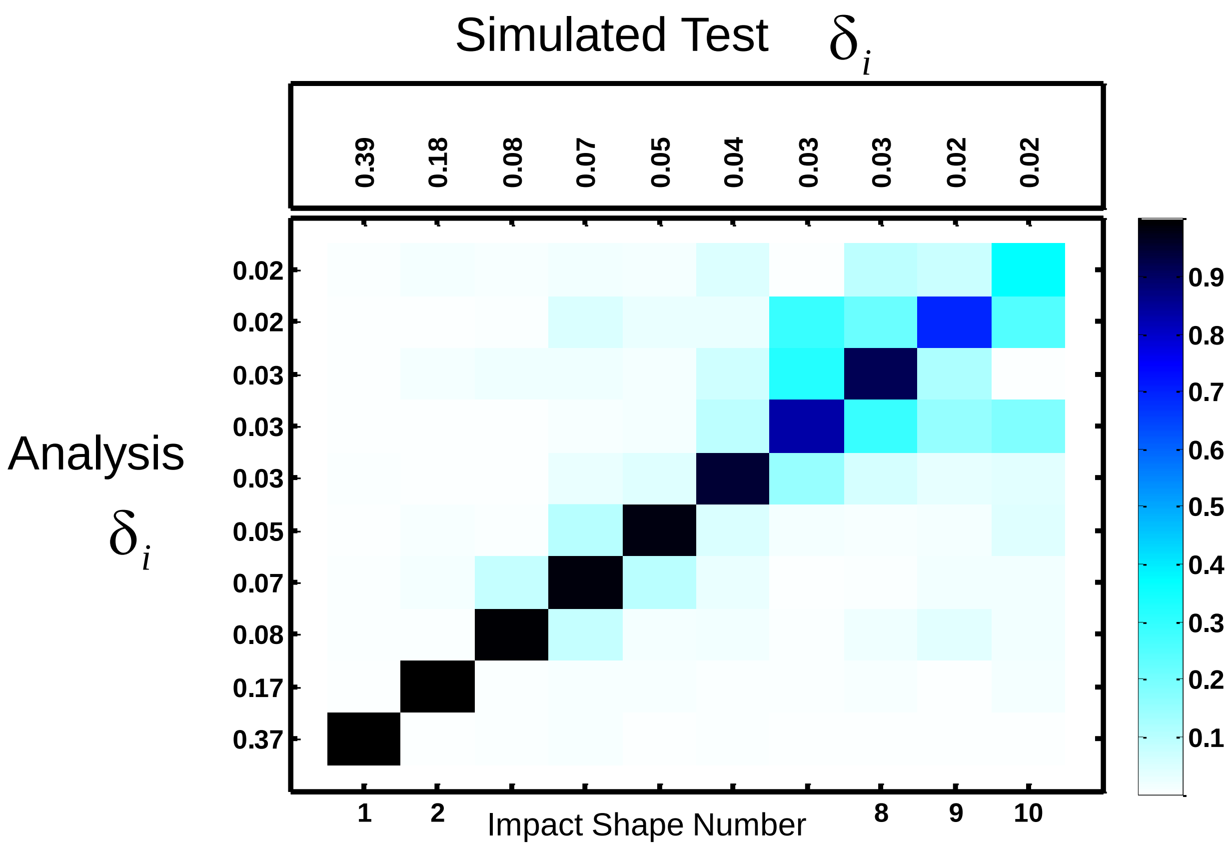
2-Norm



No.	Parameter Description	Nominal	Lower Bound	Upper Bound	Calibrated Value
1	Keel beam stiffener thickness (in)	0.020	0.015	0.025	0.0161
2	Belly panel thickness (in)	0.090	0.08	0.135	0.1008
3	Keel beam thickness (in)	0.040	0.035	0.045	0.0358
4	Lower tail thickness (in)	0.040	0.035	0.045	0.0414
5	Back panel thickness (in)	0.020	0.015	0.025	0.0166
6	Upper tail thickness (in)	0.020	0.015	0.025	0.0168

Orthogonality Results with Calibrated Parameter Selection

Metric 2



δ_i Impact shape contribution to total response

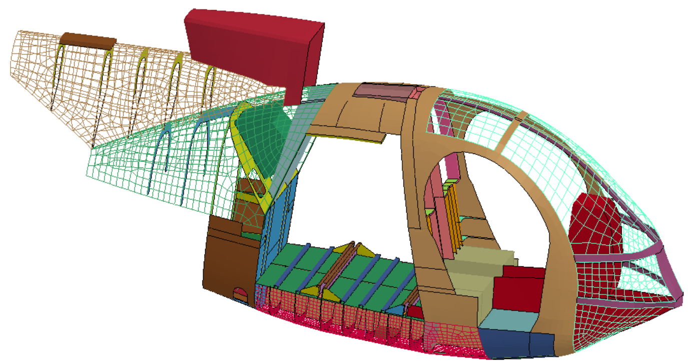
What was learned from this test?

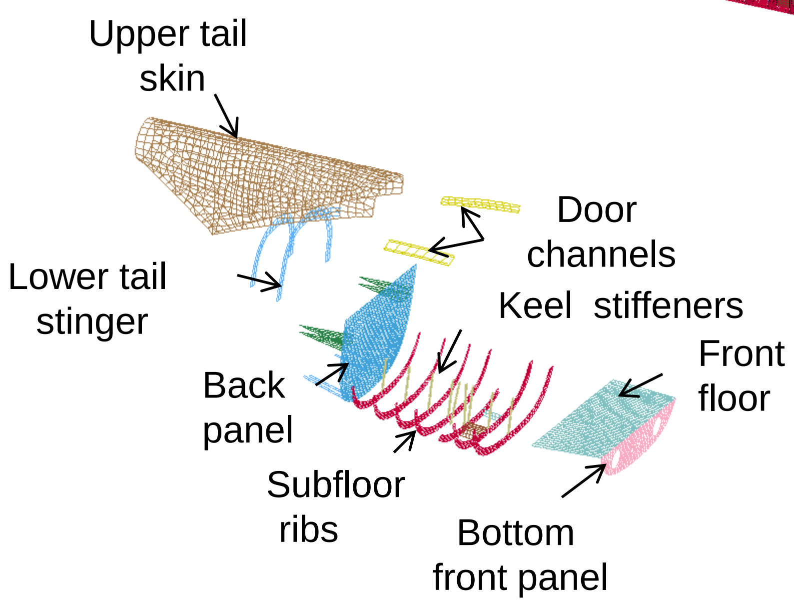
- System response predicted within anticipated uncertainty bounds
- Calibration process able to correct the analytical shape to best match simulated test
- QUESTION: Are changes in the model from the calibration correct?

ANSWER: See answer next-→

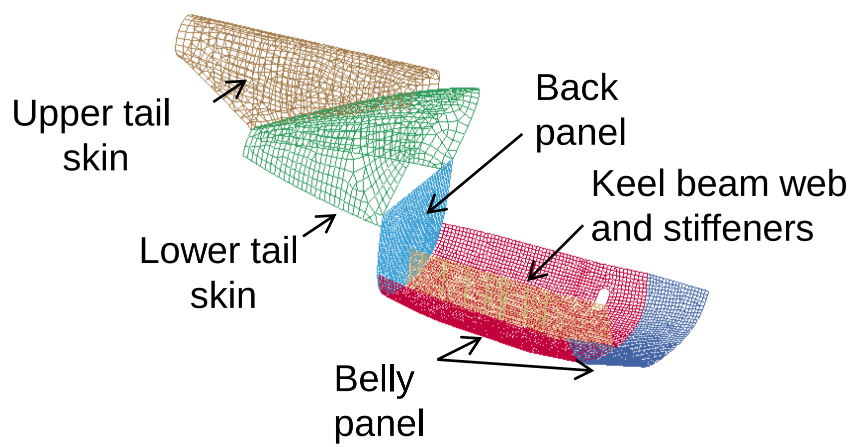
True Answer Revealed

Simplified model





Perturbed Sections



Update Sections